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The isogonal conjugate of this last point is a point whose distances from the sides are $\sin^2 a : \sin^2 b : \sin^2 c$. By repeating this process we get a series of points whose distances from the sides are as $\sin^m a : \sin^m b : \sin^m c$, where m is an even positive or negative integer. For the symmedian point we have $\sin A : \sin B : \sin C = \sin a : \sin b : \sin c$. Starting with this point and alternating as above, we get a series of points whose distances from sides are as $\sin^n a : \sin^n b : \sin^n c$ where n is any odd positive or negative integer.

NEW AND OLD PROOFS OF THE PYTHAGOREAN THEOREM.

By BENJAMIN F. YANNEY, A. M., Mount Union College, Alliance, Ohio, and JAMES A. CALDERHEAD, B. Sc., Curry University, Pittsburg, Pennsylvania.

[Continued from February Number.]

XCI. Fig. 38 and Fig. 39.

The triangle is ABC . The construction in the two figures is evident. The Hindu Bhaskara, the author of this method, complimented his readers by condensing his proof into the single word, "Behold." We follow his example.

NOTE. The above is a conjectured proof of Pythagoras. See pages 50 and 123 of Cajori's "History of Elementary Mathematics."

XCII. Fig. 38.

$$\begin{aligned} AB^2 &= 2AC \cdot BC + CH^2 \\ &= 2AC \cdot BC + (AH - AC)^2 \\ &= 2AC \cdot BC + (BC - AC)^2 \\ &= BC^2 + AC^2. \end{aligned}$$

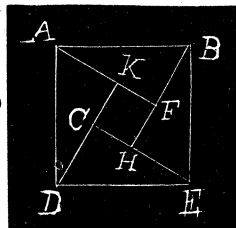


Fig. 38.

Q. E. E.

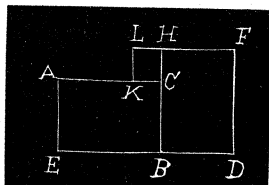


Fig. 39.

XCIII. Fig. 38.

Suppose BC is produced to meet AD , as at L . Then let LBA be the given triangle, right-angled at A .

Now, the the area of the square on AB = the sum of the four triangles ABC , BEK , DEF and ADH , and the square $CHFK$; or, $AB^2 = 2AC \cdot BC + CH^2$.

Again, $AC = (AL \cdot AB) \div BL$, $BC = AB^2 \div BL$, and $CH = AH - AC = BC - AC$.

$$\therefore 2AC \cdot BC + CH^2 = BC^2 + AC^2 = \frac{AB^4}{BL^2} + \frac{AL^2 \cdot AB^2}{BL^2}.$$

$$\therefore BL^2 = AB^2 + AL^2.$$

Q. E. D.

XCIV. Fig. 40.

$$\begin{aligned}
 AB^2 &= CF^2 - 2AC \cdot BC \\
 &= (BF + BC)^2 - 2AC \cdot BC \\
 &= (AC + BC)^2 - 2AC \cdot BC \\
 &= AC^2 + BC^2.
 \end{aligned}$$

Q. E. D.

NOTE. If we join AE in Fig. 40, and treat the trapezoid $AEEC$ after the manner of the above demonstration, we shall have a slightly different proof. This is known as General Garfield's proof. The writers are indebted, for its reproduction, to Professor Coleman Bancroft, of Hiram College, who gives also the interesting information that the proof was the result of one of Mr. Garfield's *mathematical recreations* while in Congress.

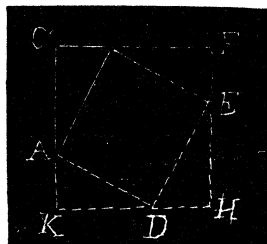


Fig. 40.

XCIV. Fig. 41. Make $BE = BC$, and draw DE perpendicular to AB . Then $DE = DC$.

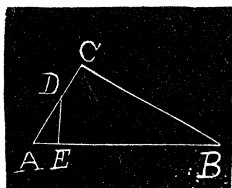


Fig. 41.

Now, the area of $ABC = \frac{1}{2}AC \cdot BC = DE \cdot BC + \frac{1}{2}AE \cdot DE$.

But $DE = (BC \cdot AE) \div AC$, and $AE = AB - BC$.

$$\therefore \frac{1}{2}AC \cdot BC = \frac{BC^2(AB - BC)}{AC} + \frac{(AB - BC)^2 BC}{AC}$$

$$\therefore AC^2 = 2AB \cdot BC - 2BC^2 + AB^2 - 2AB \cdot BC + BC^2.$$

$$\therefore AB^2 = AC^2 + BC^2. \quad \text{Q. E. D.}$$

XCVI. Fig. 41.

Let ADE be the triangle right angled at E . Produce AD to C making $DC = DE$. Produce AE till it meets the perpendicular BC drawn from C .

Then are the two triangles similar.

Now, the area of $ADE = \frac{1}{2}AE \cdot DE = \frac{1}{2}AC \cdot BC - DC \cdot BC$

$$= \frac{1}{2} \frac{DE(AD + DE)^2}{AE} - \frac{DE^2(AD + DE)}{AE}.$$

Whence, $AD^2 = AE^2 + DE^2$.

Q. E. D.

XCVII. Fig. 42.

Let ABC be the triangle, right-angled at C . Draw the escribed circle O , tangent to the hypotenuse.

Designate the sides of the triangle by a, b, c , and the radius of the circle by r .

The area of the square $CPOS = r^2 = \frac{1}{2}ab + rc$.

But $r = \frac{1}{2}(a + b + c)$.

$\therefore$ By substitution, $\frac{1}{2}(a + b + c)^2 = \frac{1}{2}ab + \frac{1}{2}c(a + b + c)$.

$$\therefore c^2 = a^2 + b^2.$$

Q. E. D.

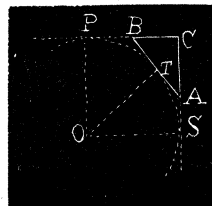


Fig. 42.

IV. QUATERNION PROOFS.

XCVIII. Fig. 43.

Represent the sides as indicated in the figure.

Then $\gamma = \alpha + \beta$. Squaring, $\gamma^2 = \alpha^2 + 2S\alpha\beta + \beta^2$.

But, since angle C is right, $2S\alpha\beta = 0$.

$\therefore \gamma^2 = \alpha^2 + \beta^2$, or as lengths simply, changing

Q. E. D.

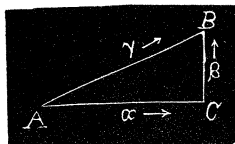


Fig. 43.

signs, $AB^2 = AC^2 + BC^2$.

From Hardy's *Elements of Quaternions*.

XCIX. Fig. 44.

Let ABC be a triangle, right-angled at C . Draw CD perpendicular to CB and equal to AC . Draw DB , which equals AB .

Represent vectors AC and CD by α , and vector CB by β .

$\therefore$ As vectors, $AB = AC + CB = \alpha + \beta$, and $DB = DC + CB = -\alpha + \beta$.

Squaring and adding, we have $AB^2 + DB^2 = 2\alpha^2 + 2\beta^2$. For the corresponding *lines*, we have $AB^2 + DB^2 = 2AC^2 + 2CB^2$.

$$\therefore AB^2 = AC^2 + CB^2.$$

Q. E. D.

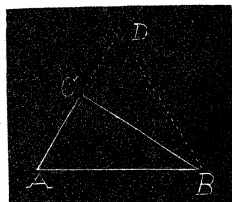


Fig. 44.

C. Fig. 45.

Let ABC be a triangle, right-angled at C . Complete the parallelogram $ADBC$. Draw the diagonal CD .

Represent vectors CB and AD by α , and vectors CA and BD by β .

$\therefore$ As vectors, $CD = CA + AD = \alpha + \beta$, and $AB = AD + DB = \alpha - \beta$.

Fig. 45. Squaring and adding, we have $CD^2 + BA^2 = 2\alpha^2 + 2\beta^2$. For the corresponding *lines*, we have $CD^2 + BA^2 = 2CA^2 + 2CB^2$.

$$\therefore 2BA^2 = 2CA^2 + 2CB^2.$$

$$\therefore AB^2 = AC^2 + CB^2.$$

Q. E. D.



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